shown that the difference between the open-loop and closed-loop characteristic equations can be used to determine the elements of the last column of the closed-loop Lyapunov matrix. These terms change the nonlinear portion of the algebraic Riccati equation to known values so that the elements of the desired state weight matrix, as well as the remaining terms of the closed-loop Lyapunov matrix, can be determined from a set of linear algebraic equations. The linearized longitudinal equations of motion of an aircraft were used to demonstrate the application of the outlined procedure.

References

[1]Steven, B. L., and Lewis, F. L., *Aircraft Control and Simulation*, Wiley-Interscience, New York, 1992, p. 341.

[2]Koussiouris, T. G., and Bakirtzis, A. G., "Optimization and Pole Placement for a Single Input Controllable System," *International Journal of Control*, Vol. 33, No. 2, 1981, pp. 355–362.

[3]Shieh, L. S., Dib, H. M., and Ganesan, A., "Linear Quadratic Regulators with Eigenvalue Placement in a Horizontal Strip," *International Journal of Systems Science*, Vol. 18, No. 7, 1987, pp. 1279–1290.

[4]Furta, K., and Kim, S. B., "Pole Placement in a Specified Disk," *IEEE Transactions on Automatic Control*, Vol. AC-32, No. 5, 1987, pp. 423–426.

[5]Arar, A. S., Sawan, M. E., and Rob, R. A., "Optimal Pole Placement in a Vertical Strip," *Proceedings of the 36th Midwest Symposium on Circuits and Systems*, Vol. 1, IEEE Circuits and Systems Society, Piscataway, NJ, 1993, pp. 653–656.

[6]Johnson, C. D., "The 'Unreachable Pole' Defect in LQR Theory: Analysis and Remedy," *International Journal of Control*, Vol. 47, No. 3, 1988, pp. 697–709.

[7]Ohta, H., and Kakinuma, C., "Use of Negative Weights in Linear Quadratic Regulator Synthesis," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 4, 1991, pp. 791–796.

[8]Al-Sunni, F. M., Stevens, B. L., and Lewis, F. L., "Negative State Weighting in the Linear Quadratic Regulator for Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1279–1281.

[9]Saif, M., "Optimal Linear Regulator Pole Placement by Weight Selection," *International Journal of Control*, Vol. 50, No. 1, 1989, pp. 399–414.

[10]Arar, A. R. S., and Sawan, M. E., "Optimal Role Placement with Prescribed Eigenvalues for Continuous Systems," *Journal of the Franklin Institute*, Vol. 330, No. 5, 1993, pp. 985–994.

[11]Shue, S. P., Sawan, M. E., and Rokhsaz, K., "Optimal Pole Placement by Weight Selection for a Single Input Controllable System," *Proceedings of AIAA Guidance, Navigation, and Control Conference* (Baltimore, MD), AIAA, Washington, DC, 1995, pp. 1142–1147 (AIAA Paper 95-3299).

[12]Roskam, J., *Airplane Flight Dynamics and Automatic Flight Control, Part I*, Roskam Aviation and Engineering Corp., Ottawa, KS, 1979, pp. 616–619.

Approximate Pole Placement for Acceleration Feedback Control of Flexible Structures

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Introduction

THE vibration control of flexible structures is a tricky problem because a finite order controller is used to control an (essentially) infinite dimensional system. Robustness is of paramount importance here because an accurate knowledge of the system is often lacking, and it is impossible to model all of the modes. Furthermore, there are several destabilizing factors, such as the effect of spillover[1] and the interaction of finite bandwidth actuator's dynamics,[2] all of which must be carefully considered when designing the controller. In Ref. 3, an active vibration control technique for second-order systems using acceleration feedback was proposed and shown to be unconditionally stable. Its robustness is further demonstrated in actual experiments[4] where single-input/single-output (SISO) controllers are designed to augment the damping of a single target mode. In a more restricted context, Sim and Lee[5] also show that after incorporating finite actuator dynamics the collocated acceleration feedback control scheme is globally and unconditionally stable. No constructive procedure, however, is proposed therein to assign desired closed-loop damping to the controlled modes.

This Note proposes a procedure for the design of a multi-input/multi-output (MIMO) controller that assigns prescribed closed-loop damping to multiple target controlled modes. We show that up to 50% closed-loop damping can be assigned to each of the controlled modes approximately, while at the same time ensuring that all the uncontrolled and unmodeled modes remain stable with higher than natural closed-loop damping.

Received June 17, 1994; revision received May 7, 1995; accepted for publication Sept. 13, 1995.

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Acceleration Feedback of a Scalar Second-Order System

To understand how the pole-placement procedure works, we first examine the acceleration feedback control of a scalar second-order system. As in the case of positive position feedback control,[2] we associate a tuning filter with sufficiently high open-loop damping to the system, and transfer some of the open-loop damping of the tuning filter to the system via an appropriate arrangement. Consider the following closed-loop arrangement of a scalar second-order system with a second-order tuning filter.

System:

$$\ddot{y} + 2\zeta_n\omega_n\dot{y} + \omega_n^2 y = -\gamma^2\eta \tag{1}$$

Tuning filter:

$$\ddot{\eta} + 2\zeta_f\omega_f\dot{\eta} + \omega_f^2\eta = \omega_f^2\ddot{y} \tag{2}$$

where $\omega_n > 0, \zeta_n > 0, \omega_f > 0$, and $\zeta_f > 0$ are the natural frequency and natural damping ratio of the system and natural frequency and natural damping ratio of the filter, respectively. Here, γ^2 is a scalar gain to be determined.

Proposition 1. The closed-loop system (1) and (2) is unconditionally stable.

Proof. This follows readily by a direct application of the Routh–Hurwitz criterion on the closed-loop characteristic equation

$$P(s) = \left(s^2 + 2\zeta_n\omega_n s + \omega_n^2\right)\left(s^2 + 2\zeta_f\omega_f s + \omega_n^2\right) + \gamma^2\omega_f^2 s^2 = 0 \tag{3}$$

As a design problem, the filter's parameters and the gain are to be determined such that a desired closed-loop damping for the system is to be achieved. We assume that the natural damping of the system is very small ($\zeta_n \ll 1$) (otherwise there is no need for active damping enhancement), hence the system's open-loop poles are close to the imaginary axis. A quick root-locus reveals that the most effective tuning filter's parameters are given by $\zeta_f = 1$ (critically damped filter) and $\omega_f = \omega_n$. The corresponding root locus diagram is as shown in Fig. 1, where ζ_n is assumed to be zero and ω_n is normalized to unity. Maximal closed-loop damping for the system is achieved at the breakaway point where the branch emanating from the system's open-loop pole meets the branch emanating from the filter's open-loop pole. The breakaway point occurs when $\gamma^2 = 1$ regardless of the value of ω_n. Since under the assumption that $\zeta_n = 0, \omega_n = \omega_f, \zeta_f = 1$, the characteristic equation reduces to

$$\left(s^2 + \omega_n^2\right)(s + \omega_n)^2 + \omega_n^2 s^2 = \left\{[s + (\omega_n/2)]^2 + \left(3\omega_n^2/4\right)\right\}^2 = 0 \tag{4}$$

hence the repeated poles are to be found at $s = -\frac{1}{2}\omega_n \pm \sqrt{(3/2)}\omega_n i$ resulting in a maximal closed-loop damping ratio of 50% for both the system and the tuning filter. Increasing γ^2 beyond unity will not increase the damping of the system or the filter; but nevertheless it will not destabilize the system either.

Once the filter parameters are tuned to the system's, it is easy to design the feedback gain to achieve a prescribed closed-loop

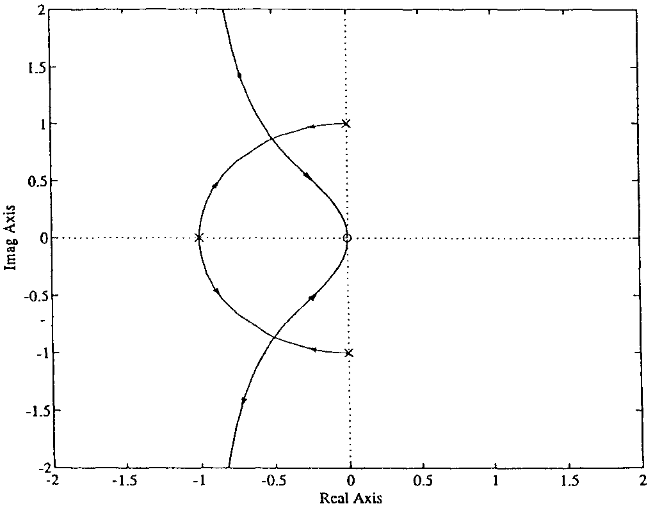

Fig. 1 Root locus of a scalar acceleration feedback control system.

damping ratio for the system. If we normalize $\omega_n = \omega_f = \zeta_f = 1$, the closed-loop damping ratios of the system (ζ_{sc}) and of the filter (ζ_{fc}) are related to the open-loop damping ratio and feedback gain by

$$(s^2 + 2\zeta_n s + 1)(s + 1)^2 + \gamma^2 s^2 = (s^2 + 2\zeta_{sc} s + 1)(s^2 + 2\zeta_{fc} s + 1) \tag{5}$$

By equating the coefficients of powers of s, we obtain

$$\gamma^2 = 4(\zeta_{sc} - \zeta_n)(1 - \zeta_{sc}) \tag{6}$$

and

$$\zeta_{fc} = 1 + \zeta_n - \zeta_{sc} \tag{7}$$

provided that $\gamma^2 \le (1 + \zeta_n)/2$. Equation (6) can be used, for any given natural frequency, to determine the feedback gain required for a prescribed closed-loop damping ratio of ζ_{sc}.

Acceleration Feedback Controller Design for a Large Second-Order System

In theory, a flexible structure is a distributed parameter hyperbolic system modeled by a generalized wave equation. In practice, however, the infinite dimensional system is discretized, via some appropriate finite element analysis, into a large but finite dimensional second-order system,

$$M\ddot{x} + D\dot{x} + Kx = f(t) \tag{8}$$

where $x \in R^N$, $N \gg 1$ denotes the physical state of the system, M, D, K are symmetric matrices in $R^{N\times N}$ representing the mass, damping, and stiffness of the structure, respectively, and $f(t)$ is some external forcing term. For collocated control, consider a set $\mathcal{S} \subset \{1, 2, \ldots, N\}$ where for each state x_j, $j \in \mathcal{S}$ we assign a pair of collocated sensor/actuator. Let $|\mathcal{S}| = n$ be the total number of sensor/actuator pairs. Furthermore, we define the sensor/actuator location matrix $S \in R^{n\times N}$ to be

$$[S]_{ij} = \begin{cases} 1 & \text{if sensor/actuator pair } i \text{ is located at state } x_j,\ j \in \mathcal{S} \\ 0 & \text{otherwise} \end{cases}$$

With the notations introduced, the sensor output may be succinctly expressed as $S\ddot{x} \in R^n$. Without loss of generality, we assume that the n pairs of sensor/actuator are used to control the first n modes of the system (with natural frequencies ω_i, $i = 1, \ldots, n$). In practice, the n pairs of sensor/actuator may be used to control more than n modes, especially when the modes' natural frequencies are closely spaced. As in the scalar case, n critically damped tuning filters with natural frequencies ω_{f_i}, $i = 1, 2, \ldots, n$ are assigned for each of the controllers, and each tuned to the frequency of the controlled modes, i.e., $\omega_{f_i} = \omega_i$. Although the actuators have finite bandwidth and, hence, should be included in the design consideration, in theory it is possible to eliminate the actuators' dynamics altogether by an appropriate arrangement as described in Ref. 2, where it is done in a different context. Henceforth we shall ignore the actuator dynamics, and close the feedback loop by the following arrangement:

Structure:

$$M\ddot{x} + D\dot{x} + Kx = -S^T C^{\frac{T}{2}} z \tag{9}$$

Tuning filter:

$$\ddot{z} + \mathcal{D}_f \dot{z} + \Omega_f z = C^{\frac{1}{2}} S\ddot{x} \tag{10}$$

where $z \in R^n$ is the state of the tuning filter, $C = C^{T/2}C^{1/2} \in R^{n\times n}$ is the (positive definite) feedback gain matrix, $\mathcal{D}_f = \operatorname{diag}\{2\omega_{f_i}, i = 1, \ldots, n\}$ is the filter's damping matrix, and $\Omega_f = \operatorname{diag}\{\omega_{f_i}^2, i = 1, \ldots, n\}$ is the filter's stiffness matrix. Next we apply the usual modal transformation to the state equation

$$x = \Phi\xi \tag{11}$$

where $\Phi \in R^{N\times N}$ is the modal matrix that simultaneously diagonalizes M and K, i.e.,

$$\Phi^T M \Phi = I_N, \qquad \Phi^T K \Phi = \Omega = \operatorname{diag}\left\{\omega_{n_i}^2, \quad i = 1, 2, \ldots, N\right\} \qquad \Phi^T D \Phi = \mathcal{D} \tag{12}$$

In the context of flexible structures, the modal damping matrix $\mathcal{D}$ is symmetric, diagonally dominant, and has small but positive eigenvalues. For convenience we shall, to first-order approximation, assume $\mathcal{D}$ to be diagonal (with diagonal elements given by $2\zeta_n\omega_i$). Note that the closed-loop stability of the system is not dependent on this assumption. Equations (9) and (10) thus transform to

$$\ddot{\xi} + \mathcal{D}\dot{\xi} + \Omega\xi = -\Phi^T S^T C^{\frac{T}{2}} z \tag{13}$$

$$\ddot{z} + \mathcal{D}_f \dot{z} + \Omega_f z = C^{\frac{1}{2}} S\Phi\ddot{\xi} \tag{14}$$

To control the first n modes using the design procedure for scalar systems as described earlier, we construct the feedback gain matrix C such that the first n controlled modes are approximately decoupled from the higher $N - n$ uncontrolled modes. To achieve this, we note that the closed-loop characteristic equation of the system (13) and (14) is given by

$$\det\left[s^2 I_N + s\mathcal{D} + \Omega + \Phi^T S^T C^{\frac{T}{2}} \times \left(s^2 I_n + \mathcal{D}_f s + \Omega_f\right)^{-1} C^{\frac{1}{2}} S\Phi s^2\right] = 0 \tag{15}$$

If the gain matrix C is designed such that

$$B = \Phi^T S^T C^{\frac{T}{2}} \left(s^2 I_n + \mathcal{D}_f s + \Omega_f\right)^{-1} C^{\frac{1}{2}} S\Phi = \begin{matrix} & n & N-n \\ n & B_{11} & B_{12} \\ N-n & B_{12}^T & B_{22} \end{matrix} \in R^{N\times N} \tag{16}$$

where

$$B_{11} = \operatorname{diag}\left\{\frac{\omega_{f_i}^2 \gamma_i^2}{s^2 + 2\omega_{f_i} s + \omega_{f_i}^2}, \quad i = 1, \ldots, n\right\} \tag{17}$$

then the first $n \times n$ subblock of the matrix

$$\Lambda = s^2 I_N + s\mathcal{D} + \Omega + \Phi^T S^T C^{\frac{T}{2}} \times \left(s^2 I_n + \mathcal{D}_f s + \Omega_f\right)^{-1} C^{\frac{1}{2}} S\Phi s^2 \tag{18}$$

becomes diagonal with diagonal entries given by

$$(\Lambda)_{ii} = s^2 + 2\zeta_n\omega_i s + \omega_i^2 + \frac{\omega_{f_i}^2 \gamma_i^2 s^2}{s^2 + 2\omega_{f_i} s + \omega_{f_i}^2} \tag{19}$$

Equation (19) implies that the first n controlled modes can be regarded as approximately decoupled scalar systems as in Eqs. (1) and (2), notwithstanding the weak coupling through B_{12} that can be ignored to first-order approximation if the γ_i^2 are sufficiently small, and the sensor/actuator locations properly chosen. In fact, as the example in the next section will show, this coupling effect remains

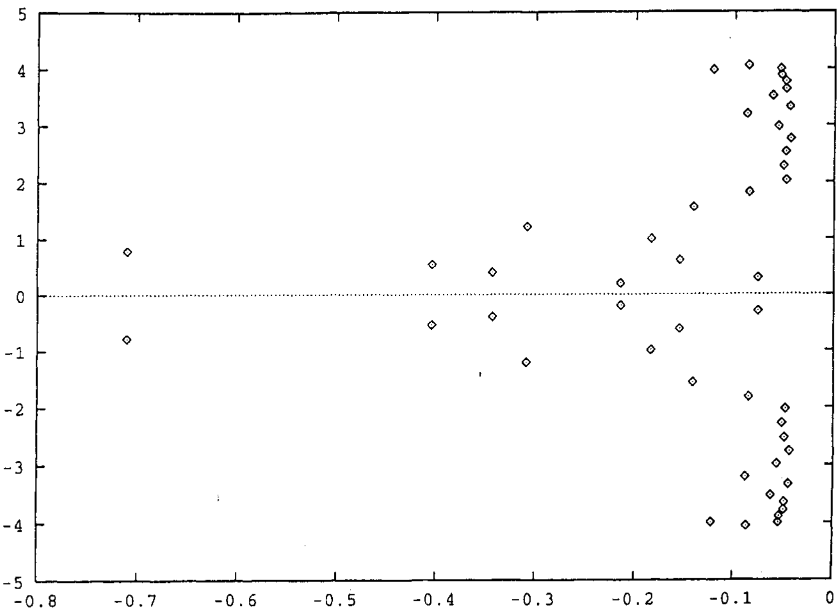

Fig. 2 Distribution of closed-loop poles with 25% prescribed damping ratio for four modes.

small even if we use $\gamma_i^2 = 1$ for maximal closed-loop damping. These scalar gains γ_i^2 can subsequently be designed to achieve the prescribed closed-loop damping for the controlled modes according to the procedure described before for scalar systems. The special structure of the B_{11} matrix as specified in Eq. (17) can be arranged by noting that

$$S\Phi = (\Psi_1 \vdots \Psi_2) \in R^{n\times N} \tag{20}$$

where $\Psi_1 \in R^{n\times n}$ and $\Psi_2 \in R^{n\times(N-n)}$. Hence,

$$B_{11} = \Psi_1^T C^{\frac{T}{2}} \left(s^2 I_n + \mathcal{D}_f s + \Omega_f\right)^{-1} C^{\frac{1}{2}} \Psi_1 \tag{21}$$

By a careful choice of the sensor/actuator location matrix S, it is possible to ensure that Ψ_1 is nonsingular. Furthermore, by locating the sensor/actuator pairs away from the nodes of the controlled modes, large entries of Ψ_1^{-1} can also be avoided, thus minimizing the coupling effect of B_{12}. Consequently, the modal gain matrix B_{11} of Eq. (21) is a result of choosing the feedback gain matrix to be such that

$$C^{\frac{1}{2}} = \mathrm{diag}\{\omega_{f_i}\gamma_i, \quad i = 1, \ldots, n\}\Psi_1^{-1} \tag{22}$$

Lastly, we note that even if the gain matrix C is not designed properly, (for example, by a bad choice of sensor/actuator locations resulting in a near-singular Ψ_1) there is no risk whatsoever that the feedback system can become unstable, since it has been shown that the closed-loop system (13) and (14) is unconditionally stable in Refs. 3 and 5.

Simulation of a Discrete Shear Beam

Consider a $N = 20$ elements discrete shear beam with $M = I_{20}$, and K defined by $K_{ii} = 2, i = 1, \ldots, N$; $K_{i,i+1} = -1, i = 1, \ldots, N-1$; $K_{i-1,i} = -1, i = 2, \ldots, N$; $K_{ij} = 0$ otherwise. It is easy to show that the natural frequencies of all N modes are given by $\omega_i = 2\sin[i\pi/(N+1)]$, and the modal matrix is given by $\Phi_{ij} = \sqrt{[2/(N+1)]}\sin[ij\pi/(N+1)]$. For convenience, we assume that the modal damping matrix is diagonal with diagonal elements given by $\mathcal{D}_{ii} = 2\zeta_n\omega_i$, where $\zeta_n = 0.01$ is assumed to be uniform for all modes. To control the first four modes $n = 4$ sensor/actuator pairs are used, and we select the sensor/actuator pairs locations to correspond to $\mathcal{S} = \{3, 8, 13, 19\}$. Four critically damped tuning filters with natural frequencies tuned to their respective controlled modes will be used to create the desired damping.

The resulting closed-loop system is a second-order system with $N + n = 24$ states. If we prescribed a closed-loop damping ratio of 25% for the first four controlled modes, the design procedure suggests the use of $\gamma_i^2 = 0.72, i = 1, 2, 3, 4$. The resulting closed-loop eigenvalues are as shown in Fig. 2, where the actual closed-loop damping ratio of the first four modes are 0.252, 0.245, 0.182, and 0.248, respectively. With the exception of the third mode, the actual closed-loop damping ratios are within 2% of the design values. The relatively low closed-loop damping of the third mode is probably because three out of four of the sensor/actuator pairs are placed near the nodes of the third mode, thus diminishing their influence. Furthermore, all of the uncontrolled modes (and, expectedly, the unmodeled modes) result in higher than natural damping. Further design for a maximal damping ratio of 50% result in the actual closed-loop damping ratio of 0.547, 0.54, 0.433, and 0.523 for the first four controlled modes. Despite the large gains used, the coupling effect of the uncontrolled modes remains minimal. Again all uncontrolled modes result in higher than natural damping.

Concluding Remarks

There remains a couple of practical issues that require further attention. First, accelerometers admit some delay in their measurement of acceleration. This delay effectively introduces open-loop zeros in the right-half complex plane and may cause eventual instability when the feedback gain is large enough. Because of the extreme robustness of the technique, however, some preliminary simulation studies have shown that the closed-loop system remains stable well beyond the design range of the gain, in spite of the presence of fairly large time delay. Second, accelerometers measure absolute (and not relative) acceleration, which makes it impossible to decouple the rigid body mode from the vibration modes of the flexible structure. Studies are underway to address these issues.

Acknowledgments

This research was supported by a grant from the Australian Research Council. We thank T. K. Caughey for many helpful comments.

References

[1]Balas, M. J., "Active Control of Flexible Structures," *Journal of Optimization Theory and Applications*, Vol. 25, 1978, pp. 415–436.

[2]Goh, C. J., and Caughey, T. K., "On the Stability Problem Caused by Finite Actuator Dynamics in the Collocated Control of Large Space Structures," *International Journal of Control*, Vol. 41, 1985, pp. 787–802.

[3]Juang, J. N., and Phan, M., "Robust Controller Designs for Second-Order Dynamic Systems: A Virtual Passive Approach," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1192–1198.

[4]Bruner, A. M., Belvin, W. K., Horta, L. G., and Juang, J. N., "Active Vibration Absorber for the CSI Evolutionary Model: Design and Experimental Results," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1253–1257.

[5]Sim, E., and Lee, S. W., "Active Vibration Control of Flexible Structures with Acceleration Feedback," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 413–415.

Nearly Nonminimal Linear-Quadratic-Gaussian Compensators for Reduced-Order Control Design Initialization

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I. Introduction

THE development of linear-quadratic-Gaussian (LQG) theory was a major breakthrough in modern control theory since it provides a systematic way to synthesize high-performance controllers for nominal models of complex, multi-input multi-output systems. One of the well-known deficiencies of an LQG compensator, however, is that its minimal dimension is usually equal to the dimension of the design plant. This has led to the development of techniques to synthesize reduced-order approximations of the optimal full-order compensator (see Refs. 1–3 and the references therein).

The controller reduction methods almost always yield suboptimal (and sometimes destabilizing) reduced-order control laws since an optimal reduced-order controller is not usually a direct function of the parameters used to compute or describe the optimal full-order controller. Nevertheless, these methods are computationally inexpensive and sometimes do yield high performing and even nearly optimal control laws. An observation that holds true about most of these methods is that they tend to work best at low control authority.[2,3] To date, however, no algebraic conditions have been established that guarantee that a given suboptimal controller reduction method will work well at low authority.

This Note considers the balanced controller reduction algorithm of Ref. 1 and provides a constructive way of choosing the weights in an LQG control problem of dimension n such that for a given $n_c < n$ the corresponding n_cth-order controller obtained by this suboptimal reduction method has essentially the same performance as the LQG controller at low control authority. The usefulness of this result is for initializing homotopy algorithms for optimal reduced-order control design that requires a nearly optimal controller for an initial set of design parameters.[4]

The discussion here focuses on stable systems. It is shown that if the state weighting matrix R_1 or disturbance intensity V_1 has a specific structure in a basis in which the plant dynamics A is upper or lower block triangular, respectively, then at low control authority the corresponding LQG compensator is nearly nonminimal with minimal dimension n_c. It follows that the LQG compensator can be easily reduced to a n_cth-order controller having nearly the same performance. These results are directly applicable in initializing continuation and homotopy algorithms[4] that require a reduced-order controller that is nearly optimal for some set of initial design parameters corresponding to a low authority controller.

A special case of the conditions presented for R_1 and V_1 has a strong physical interpretation for structural control problems. In particular, assuming that all of the eigenvalues in the plant are complex (lightly damped structures) and n_c is an even number, then either R_1 is allowed to weight only $n_c/2$ structural modes or V_1 is allowed to disturb only $n_c/2$ structural modes.

II. Construction of Nearly Nonminimal LQG Compensators

Consider the nth-order linear time-invariant plant

$$\dot{x}(t) = Ax(t)+Bu(t)+D_1w(t), \qquad y(t) = Cx(t)+D_2w(t) \quad (1)$$

where (A, B) is stabilizable, (A, C) is detectable, $x \in R^n$, $u \in R^m$, $y \in R^l$, and $w \in R^d$ is a standard white noise disturbance with intensity I_d and rank $D_2 = l$. The intensities of $D_1w(t)$ and $D_2w(t)$ are thus given, by $V_1 \triangleq D_1D_1^T \geq 0$ and $V_2 \triangleq D_2D_2^T > 0$, respectively. Then, the LQG compensator

$$\dot{x}_c(t) = A_cx_c(t) + B_cy(t), \qquad u(t) = -C_cx_c(t) \quad (2)$$

for the plant (1) minimizing the steady-state quadratic performance criterion

$$J(A_c, B_c, C_c) \triangleq \lim_{t\to\infty} \frac{1}{t} E \int_0^t \left[x^T(s)R_1x(s) + u^T(s)R_2u(s)\right] \mathrm{d}s \quad (3)$$

where $R_1 \geq 0$ and $R_2 > 0$ are the weighting matrices for the controlled states and controller input, respectively, the plant is given by

$$A_c = A - \Sigma P - Q\bar{\Sigma}, \qquad B_c = QC^TV_2^{-1}, \qquad C_c = R_2^{-1}B^TP \quad (4)$$

where $\Sigma \triangleq BR_2^{-1}B^T$ and $\bar{\Sigma} \triangleq C^TV_2^{-1}C$, and P and Q are the unique, nonnegative-definite solutions of

$$A^TP + PA + R_1 - P\Sigma P = 0 \quad (5)$$

and

$$AQ + QA^T + V_1 - Q\bar{\Sigma}Q = 0 \quad (6)$$

respectively. Furthermore, the shifted observability and controllability Gramians[1] of the compensator, $\hat{P}$ and $\hat{Q}$, are the unique, nonnegative-definite solutions of

$$(A - Q\bar{\Sigma})^T\hat{P} + \hat{P}(A - Q\bar{\Sigma}) + P\Sigma P = 0 \quad (7)$$

and

$$(A - \Sigma P)\hat{Q} + \hat{Q}(A - \Sigma P)^T + Q\bar{\Sigma}Q = 0 \quad (8)$$

respectively. The magnitudes of R_2 and V_2 relative to the state weighting matrix R_1 and plant disturbance intensity V_1 govern the regulator and estimator authorities, respectively. The selection of R_2 and V_2 such that $\|R_2\| \gg \|R_1\|$ or $\|V_2\| \gg \|V_1\|$ yields a low authority compensator. This section shows that when the open-loop plant is stable and (A, R_1) or (A, V_1) have a particular structure, the LQG controller approaches nonminimality as the controller authority decreases. To prove this result, we exploit structural properties of the solutions of Riccati and Lyapunov equations assuming the coefficient matrix A and the constant driving term R_1 have certain partitioned forms.

Lemma 2.1. Suppose

$$A = \begin{bmatrix} A_1 & 0 \\ A_{21} & A_2 \end{bmatrix}, \qquad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad R_1 = \begin{bmatrix} R_{1,1} & 0 \\ 0 & 0_{n-n_r} \end{bmatrix} \quad (9)$$

where $A_1, R_{1,1} \in R^{n_r\times n_r}$, $B_1 \in R^{n_r\times m}$, and $R_{1,1} > 0$, and assume A is asymptotically stable. Then the following statements hold.

Received April 20, 1995; revision received Aug. 25, 1995; accepted for publication Sept. 18, 1995.

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